

In Chapters 4 & 5 of *Atoms of Light*, time dilation is shown to be a logical consequence of the cyclic energy flows forming a moving particle of matter in a limited scenario., *I.e.* considering formative energy-flow as a simple circle perpendicular to line of motion.

This appendix extends that finding to a generalised cyclic energy-flow pattern

Chapter 5 may be downloaded from <https://r.ihs.ac/ch5.pdf> , for reference.

## Appendix A

### **The Spun-Light\* Mathematics of Time Dilation**

(\* aka ‘cyclic-photon’)

## How Many Ways Can You Loop the Loop?

In Chapter 4 we looked at the effects on time-perception of movement for a particle that was made up of a single loop of energy flow repeatedly following a simple circular path. We now need to open up the options quite a bit, to allow for **any** repeating cyclic energy pattern, or combination of such patterns, that might compose a material particle (or multi-particle object).

This would include: loops that might follow an irregular path before coming back to where they started; loops that don't quite join up first time around but take a second circuit, or maybe more, before catching up with themselves; loops that form a figure of eight or any other exotic shape. Any flow-path is a possibility that needs to be considered, as long as it forms a closed path; otherwise we haven't got anything we can call a 'particle' <sup>1</sup>.

[Note that the closed path becomes a very ornate spiral when the particle is on the move – that's part of the deal].

We must even consider the possibility that the flow pattern making up a particle may actually comprise a number of cyclic energy flows working together. In this case we can establish a result for each of those flows separately and then show that the result holds for the group. In the same way an energy flow that's spread across a surface, for example the surface of a sphere, and repeatedly flows around that surface can be treated as a vast collection of individual threads of energy (using the concept of infinitesimals) that each follow their own repeating path.

We'll now focus on just one thread of energy flow making up the particle. It may combine with other flows to form the particle, it may be the only flow that's needed for the formation of the particle; that doesn't matter. What we do know is that this one thread of flow forms a closed loop, however simple or complex it may be.

As we've already established, if a particle is moving then the energy flows that make up that particle are doing two things. They're cycling round a repeating pattern that makes up the particle itself, and they're travelling through space in the direction that the particle is moving; that linear energy flow *is* the movement of the particle.

From this description we can form a *vector triangle* showing the relationship at any given instant between cyclic energy flow (forming the particle), linear flow (moving the particle through space) and the overall energy flow in the particle that gives rise to these two effects. That relationship will vary as the cyclic element of flow traces out its repeating path.

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1. It might be argued that a particle's formative energy-flow may follow a non-repeating path, like a 3D equivalent of an irrational number such as pi; this seems most improbable. However, even if this were the case it would still be true that, for any starting point, that energy-flow would at irregular intervals re-approach arbitrarily close to that starting point. This satisfies the requirements for a 'loop' in this proof.

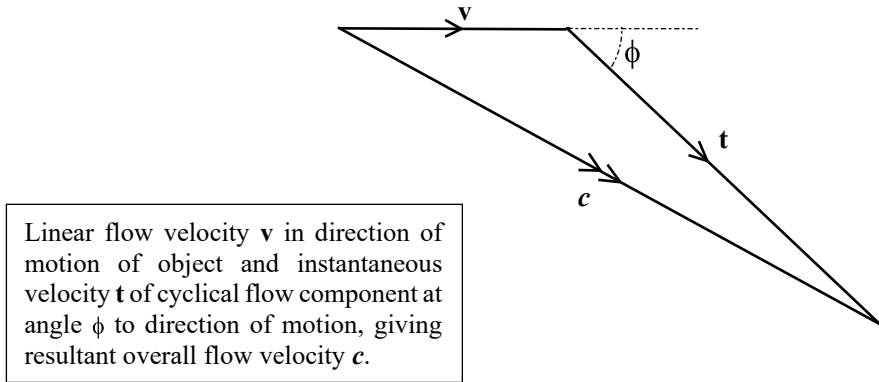
(A) Detailed spun-light mathematics of: Time Dilation.

(Note: **bold** indicates vectors, non-bold magnitudes only, throughout below.)

The linear flow, or particle velocity, component is labelled **v** in the diagram below. The cyclic flow velocity component, as it's responsible for the time-experience of the particle, is labelled **t**. The overall flow, which is the *resultant* made up of these two components, is labelled **c** (since it's a photon energy flow).

In a way that's putting things back to front, since the fundamental reality is the energy flow moving at constant speed **c**. This gives rise to the formation of the particle itself and also to the movement of that particle through space at velocity **v**. That spatial velocity component will be constant for the whole cycle, both in magnitude and direction – the speed and direction of motion of the particle.

At any given instant the cyclic component **t** will be in the direction that the cyclic flow is moving at that instant, of a magnitude that combines with **v** to give a resultant equal in size to the overall flow speed **c**. In other words both the magnitude and direction of **t** will vary throughout each cycle<sup>2</sup>.



Applying the Cosine Rule to the lengths in this triangle, we get:

$$\begin{aligned} c^2 &= v^2 + t^2 - 2 v t \cos(180^\circ - \phi) \\ &= v^2 + t^2 + 2 v t \cos \phi \\ \Rightarrow t^2 + 2 v t \cos \phi + (v^2 - c^2) &= 0 \end{aligned}$$

Solving this quadratic equation for **t** gives us:

$$t = \sqrt{(c^2 - v^2 \sin^2 \phi)} - v \cos \phi \quad [\text{as } 1 - \cos^2 \phi = \sin^2 \phi]$$

where **t** is the speed of the cyclic component of energy flow at a point in its cycle where it is at angle  $\phi$  to the direction of particle motion.

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2. This is effectively a vector subtraction: the cyclic component **t** is the full energy flow **c** less the linear motion component **v**. The direction of **c** is determined by **v** and the direction at that instant of the cyclic flow **t**.

(A) Detailed spun-light mathematics of: Time Dilation.

The purpose of this exercise is to establish whether the time dilation factor identified in Chapter 4 for a simple circular flow also holds for all more complex cyclic flow patterns.

As a reminder, that relationship was:

$$(\text{cyclic speed})^2 + (\text{linear speed})^2 = (\text{flow speed})^2$$

In terms of the letters used on our diagram on the previous page, this becomes:

$$t^2 + v^2 = c^2$$

or:

$$t^2 = c^2 - v^2$$

In this more complex situation  $t$  varies as the cyclic flow component performs its periodic loop. So we need to consider the *average* value of  $t^2$  for a complete cycle – what's called the *mean square* value.

We do this as follows:

Imagine yourself being inside the particle (you'll have to be quite small for this!), that is, within the loop made by the cyclic energy flow. Now imagine yourself holding out your hand so that your index finger touches one point on that repeating loop. Since you're travelling at the same speed as the particle, that energy flow will repeatedly touch your finger, once each circuit – your own motion will exactly make up for the gap between successive circuits (the 'spiral' effect) that's due to the linear motion of the particle.

In mathematical terms we refer to this loop that touches your finger, goes off on whatever circuit it wants, then comes back again to touch your finger, as a *contour* – a closed loop, which is what you'd see it as. We use a *contour integral* to calculate the average value of  $t^2$  for that closed loop. Notice that in this way we've eliminated the linear particle motion from our calculation, whilst still using values of  $t$  that take account of that linear motion.

If you haven't met contour integrals before, don't let that bother you. It's simply a way of adding up a very large number of very small bits, just like any other integral except that these bits are arranged around a closed loop. In this particular case we don't even need to do any integration!

On the previous page we established the formula for cyclic speed  $t$  at any instant:

$$t = \sqrt{(c^2 - v^2 \sin^2 \phi)} - v \cos \phi$$

where  $\phi$  is the angle between cyclic flow and particle motion at that instant.

This gives us:

$$\begin{aligned} t^2 &= [\sqrt{(c^2 - v^2 \sin^2 \phi)} - v \cos \phi]^2 \\ &= c^2 - v^2 \sin^2 \phi + v^2 \cos^2 \phi - 2 v \cos \phi \sqrt{(c^2 - v^2 \sin^2 \phi)} \\ &= c^2 - v^2 + 2 v^2 \cos^2 \phi - 2 v \cos \phi \sqrt{(c^2 - v^2 \sin^2 \phi)} \end{aligned}$$

$$[\text{Since } -v^2 \sin^2 \phi = -v^2(\sin^2 \phi + \cos^2 \phi) + v^2 \cos^2 \phi = -v^2 + v^2 \cos^2 \phi]$$

(A) Detailed spun-light mathematics of: Time Dilation.

To find the average  $t^2$  we simply go round our contour multiplying each different value of  $t^2$  by the time that it acted for (**remembering that  $t$  is a speed**) and then divide the total of those values ( $t^2 \times$  time of action) by the total time for the cycle. The effect of the contour integral is to divide the complete loop up into infinitesimally small segments each with its own associated speed of flow,  $t$ .

***Don't worry, it drops out very easily!***

So we're simply adding together all those infinitesimal amounts of: ( $t^2 \times$  tiny bits of time), then dividing by the sum of all those tiny intervals of time – which is of course the total time for going round the loop – to give the average  $t^2$  around that loop.

Our contour integral is represented by the symbol  $\oint$ . Each infinitesimal length of the contour (loop) is represented by  $dc$ .

[Note that 'dc' in this context has nothing to do with light speed  $c$ .]

If the cyclic component of energy flow moves along a section length  $dc$  of the loop contour at speed  $t$ , then of course the time taken for that section is:

$$\frac{dc}{t} \quad \left[ \text{This follows very simply from the principle that time} = \frac{\text{distance}}{\text{speed}} \right]$$

Three things follow from this:

- (i) Total time for travel around loop =  $\oint \frac{dc}{t}$   
(The sum of all those infinitesimal bits of time)
- (ii) The effect of  $t^2$  acting over the time taken for segment  $dc = t^2 \times \frac{dc}{t}$
- (iii) The cumulative total for ( $t^2 \times$  time) acting around the complete contour is the sum of all *those* infinitesimal bits:  $\oint t^2 \frac{dc}{t}$

We're now ready to calculate **the average of  $t^2$  over one cycle**. This is:

$$\frac{\text{sum of } (t^2 \times \text{time}) \text{ for all segments of the cyclic path}}{\text{total time for cyclic path (i.e. around the loop)}} = \frac{\oint t^2 \frac{dc}{t}}{\oint \frac{dc}{t}}$$

Substituting in the expression for  $t^2$  from the bottom of the previous page we get:

$$\text{average } t^2 = \frac{\oint [c^2 - v^2 + 2 v^2 \cos^2 \phi - 2 v \cos \phi \sqrt{(c^2 - v^2 \sin^2 \phi)}] \frac{dc}{t}}{\oint \frac{dc}{t}}$$

On the following page we'll re-state this equation and simplify it down. This will give us the *mean square* speed of the cyclic component of energy flow around a particle – which in turn relates particle speed to time-perception.

(A) Detailed spun-light mathematics of: Time Dilation.

On the last page we established that:

$$\begin{aligned} \text{average } t^2 &= \frac{\oint [c^2 - v^2 + 2 v^2 \cos^2 \phi - 2 v \cos \phi \sqrt{(c^2 - v^2 \sin^2 \phi)}] \frac{dc}{t}}{\oint \frac{dc}{t}} \\ &= \frac{\oint (c^2 - v^2) \frac{dc}{t} - \oint 2v \cos \phi [\sqrt{(c^2 - v^2 \sin^2 \phi)} - v \cos \phi] \frac{dc}{t}}{\oint \frac{dc}{t}} \end{aligned}$$

(Simply splitting the top contour integral into two parts and taking a common factor  $-2v \cos \phi$  out of the second part.)

Since  $c$  and  $v$  are both constants (overall energy flow speed and speed of particle motion) then the term  $(c^2 - v^2)$  can be taken outside the first integral as a constant multiple. Also the expression in square brackets in the second integral corresponds to the expression we derived for  $t$  from our vector triangle three pages back, so we can replace that whole square bracket term with just  $t$ .

This gives us:

$$\begin{aligned} \text{average } t^2 &= \frac{(c^2 - v^2) \oint \frac{dc}{t} - \oint 2v \cos \phi t \frac{dc}{t}}{\oint \frac{dc}{t}} \\ &= (c^2 - v^2) - \frac{2v \oint \cos \phi dc}{\oint \frac{dc}{t}} \quad \begin{array}{l} \text{[Cancelling equal contour} \\ \text{integrals on top and bottom} \\ \text{of first part of the fraction.]} \end{array} \end{aligned}$$

[Constant term  $2v$  comes outside second top integral,  $t$  cancels in that integral.]

Now ' $\cos \phi dc$ ' is simply a rearrangement of ' $dc \cos \phi$ ', where  $dc$  is an infinitesimal section of the contour (cyclic flow loop) for cyclic flow moving at speed  $t$  at an angle  $\phi$  to the direction of motion of the particle (see vector triangle three pages back). This is the component (or projection, whichever you prefer) of  $dc$  in the direction of particle motion  $v$ .

Bearing in mind that this contour represents **only** the cyclic motion, with the linear motion taken out of the equation, and that cyclic motion by definition always comes back to where it started (remember that flow that touches your fingertip, goes off round its loop, then comes back to your fingertip), then the net displacement of that contour in one complete cycle is zero. It follows, then, that those components/projections of  $dc$  in the direction of  $v$  must also add up to zero - the positive parts exactly balance the negative parts.

[In the hypothetical case of a non-repeating path, this net displacement may be made arbitrarily small, *i.e.* vanishingly small *c.f.* near-looping path length.]

(A) Detailed spun-light mathematics of: Time Dilation.

So that contour integral,  $\oint \cos \phi \, dc$ , in the final line of working on the previous page, gives a total of zero. The denominator of that term represents the time taken for one complete cycle around that contour, which must be greater than zero.

So the overall value of that term,  $\frac{2v \oint \cos \phi \, dc}{\oint \frac{dc}{t}}$ , reduces to zero.

This leaves us with the result:

$$\text{average } t^2 = c^2 - v^2$$

This is exactly the result we established for a simple circular cycle in Chapter 4.  $t$  is the speed of the cyclic component of energy flow, which relates directly to the rate of time experience for any particle or object. This result tells us that  $t^2$ ,  $v^2$  and  $c^2$  are related by the same mathematical relationship as the three sides of a right-angled triangle (where  $t^2$  is now averaged for the cyclic energy flow):  $t^2 + v^2 = c^2$  [Equation R, referred to below]

In other words: ***No matter how complex the cyclic energy flows that form a particle or object, every thread of energy making up those cycles – and so the totality of cyclic flow components for any object – varies in (effective\*) speed with the speed of that object’s motion in the same way that one shorter side of a right-angled triangle varies with the other for a fixed length of hypotenuse,  $c$ , regardless of direction of motion.*** [\* i.e. RMS (root mean square) speed.]

This confirms the result for the time-experience reduction factor first established in Chapter 4¾ for a simple circular energy flow, and shows that it holds for **all** material particles and objects in every direction of motion, no matter how complex their constituent cyclic energy flows:

$$\text{Time-experience reduction factor} = \frac{\text{cyclic speed}}{\text{flow speed}} = \sqrt{1 - \left( \frac{\text{linear speed}}{\text{flow speed}} \right)^2}$$

or, in the terms used to establish this result:

$$\text{Time-experience reduction factor} = \sqrt{1 - \left( \frac{v}{c} \right)^2} \quad [t/c \text{ as given by equation R above.}]$$

This exactly matches Einstein’s expression<sup>3</sup> for *relativistic time dilation*: as the speed of a particle or object increases, the time-experience of that particle/object slows down according to the above formula. This analysis shows that this is *not* due to some unique property of light but simply to the fact that all matter is formed from (possibly complex) loops of electromagnetic energy – spun light.

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3. This factor is generally used in its inverted form in Relativity:  $1/\sqrt{1 - v^2/c^2}$ , to show the *increased* duration of each second (hour, etc) measured in a moving frame as compared with a static frame. Here we have derived the *reduced* amount of time-experience in a moving frame for each unit of time that passes in the static frame.

(A) Detailed spun-light mathematics of: Time Dilation.

In the meantime we can note another very significant feature of this result that sheds more light on Einstein's interpretation of relativistic effects (effects observed at very high speeds).

We've seen that as an object's speed increases then its rate of experience of time corresponds to the side of a right-angled triangle perpendicular to the side representing the speed of motion of that object through space (where the hypotenuse is the full energy-flow speed,  $c$ ). This is true no matter what direction that object may be travelling in 3-dimensional space.

In other words, the 'direction' of time experience for any object (in a purely mathematical sense) is perpendicular to every spatial direction. For this reason time is seen in Relativity Theory as a fourth dimension, perpendicular to the three spatial dimensions. This has proved very convenient for calculations related to 4-dimensional 'spacetime' (but it's taking things too far to conclude from this mathematical relationship that time is an *actual* 4<sup>th</sup> dimension).

As observed in Chapter 7, it's worth noting that those 'spacetime' calculations in Relativity cast time in the role of an 'imaginary dimension', multiplying every measurement in the 'time direction' by the term  $i$ , the square root of minus one. Numbers involving this term are referred to in maths as *imaginary* numbers.

This reflects the simple fact that the notion of objects moving through that fourth dimension is in a sense inside-out: rather than objects moving through time, time moves through objects in the form of cyclic energy flows. Those energy flows are simply the photons of electromagnetic energy – light – that are now known to be the basis of every elementary particle of matter, and that thus also carry all time effects around every physical object.

Similarly, spacetime calculations require that any time component is multiplied by the factor  $c$  to produce correct results (either explicitly or by adjusting the time-scale of the calculation). More direct confirmation that time moves quite literally at the speed of light would be difficult to imagine; no further proof is needed, surely, that effects of time are indeed carried through matter by the energy flows forming that matter – moving at the speed of light.





This appendix builds on the concept introduced in Chapter 5 of *Atoms of Light*, where it is shown that perceived speed of light is invariant in specific situations.

That chapter may be downloaded from <https://r.ihs.ac/ch5.pdf>, for reference.

## Appendix B

### **Quasiluminal: The Spun-Light\* Mathematics of Speed-of-Light ‘Invariance’**

(\* aka ‘cyclic-photon’)

Constancy of subjective speed of light (for *any* angle of approach).

In Chapter 5 it was established that light travelling in the same direction as any object, or in the opposite direction, will appear to be moving at speed  $c$  relative to that object regardless of the object’s own speed.

We’re now going to extend that result to light travelling at *any* angle relative to the direction of motion of that object: *i.e.* light interacting with any object will appear to be moving at the speed of light,  $c$ , relative to that object, irrespective of the speed and direction of that object.

### Terms of reference

The maths on the following pages refer to an object moving with speed  $v$  in direction  $x$ . Light-flows travelling at speed  $c$  at every angle to this direction are considered for a 2-dimensional plane, using a reference axis  $y$  perpendicular to  $x$ . This covers every possible situation of experience by a moving object of light motion at true speed  $c$ .

Before we even start to consider the effects of cyclic energy flows we must first take account of the steady spatial displacement of our observer and the effect this will have on the situation.

As a simple illustration, imagine that you're driving across the desert in a West-East direction. Another vehicle up ahead of you is travelling across your path, North to South, in such a way that you're all set for a collision at the crossroads (if there *are* any crossroads in the desert).

From your point of view that other vehicle is approaching you diagonally in a Southwesterly direction, basically because you're moving steadily closer to the line of motion of that vehicle (if you weren't there would be no possibility of a collision). This isn't a subjective impression, it's an objective fact: the motion of that other vehicle relative to your ever-changing position *is* along a diagonal Southwesterly line<sup>1</sup>.

The component of the velocity of that other vehicle in the direction of this 'line of approach' is the projection of its velocity onto that line.

Note that this is *not* the velocity of that other vehicle relative to yours, it's the component of that vehicle's *actual* velocity in your direction – from the viewpoint of a *static* observer<sup>2</sup>. This is the effective speed, from a static viewpoint, that will impact on the cyclic energy flows of the observer, in this case you.

The approaching 'vehicle' in our analysis overleaf is a photon moving at speed  $c$ , shown in the diagram as approaching from various angles. That diagram and supporting maths are laid out lengthways on that double-page spread for clarity.

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1. Mariners familiar with the hazard of another vessel on a constant bearing, warning of an impending collision, will understand what I'm talking about here.

For those who remain unconvinced that this is an objective reality rather than a subjective impression, consider the following:

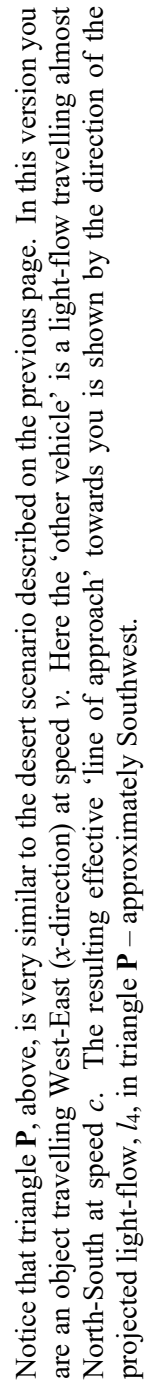
A static observer is asked to note the displacement of the other vehicle relative to yours, measuring both North and East, at one-second intervals. Those displacements are then plotted on West-East/South-North axes.

What direction will that series of points show – as recorded by a *static* observer?

2. The difference between this *actual* component of speed and the *relative* speed of the other vehicle in relation to yours can be simply demonstrated as follows:

Imagine that you are travelling at 100 m.p.h. but they are doing just 5 m.p.h. (they are much closer to the crossroads than you are). Their speed relative to you will be pretty much equal and opposite to your speed relative to them - just over 100 miles an hour. But the component of their speed along the line of approach joining the two of you cannot, by definition, exceed 5 miles an hour.

Actual light-flows  $c$ , and displaced effective flows  $l$ , as impacting on an object travelling at velocity  $v$  in direction  $x$  (shown for light-flows in the  $x$ - $y$  spatial plane).



(B) Detailed spun-light mathematics of: Perceived Invariance of Light Speed.

Figure B1, above, shows light-flows interacting with an object moving in direction  $x$  from different angles of incidence in the  $x$ - $y$  plane, representing fully all such possible situations. As explained on the previous page, the object's own speed  $v$  causes those flows to effectively be projected onto a 'line of approach' as shown in Figure B1.

Projected flow  $l$  is shown for five sample light-flow directions, including flow in both directions along the  $x$ -axis (as already considered in Chapter 5) and flow experienced as travelling along the  $y$ -axis. The projection direction is given by moving back a 'distance' (speed)  $v$  from the end of each light-flow  $c$  (*i.e.* vector subtraction). The projected length (*i.e.* effective speed)  $l$  is found by dropping a perpendicular from the end of  $c$  onto the projection direction.

[N.B. The light-flows in both directions along the  $x$ -axis follow this principle, each projection being the same as the light-flow itself. This isn't immediately apparent from the above diagram but can be confirmed by sketching a copy of that diagram with light-flows a few degrees above and below the  $x$ -axis in each direction, then following the above steps. This isn't included in Figure B1 so as not to clutter the diagram.]

As the direction of light-flow sweeps round anti-clockwise from the direction of object motion, the associated angle  $\alpha$  between its projection  $l$  and object motion,  $v$ , goes from zero for  $l_1$  to around  $70^\circ$  for  $l_2$ , through  $180^\circ$  for  $l_3$ , about  $220^\circ$  for  $l_4$  and  $270^\circ$  for  $l_5$ , back to zero ( $360^\circ$ ) as we return to the  $x$ -axis.

For each value of  $\alpha$  the value of  $l$  is given by:

$$l^2 = c^2 - (v \sin \alpha)^2 \quad [\text{Pythagoras - see example Q above}] \\ = c^2 \cos^2 \alpha + (c^2 - v^2) \sin^2 \alpha \quad [\text{using } \sin^2 \alpha + \cos^2 \alpha = 1]$$

This is the equation of an ellipse with semi-major axis  $c$  and semi-minor axis  $\sqrt{c^2 - v^2}$ . In other words the lengths of the projected light-flows for every possible angle of incidence trace out just such an ellipse.

This clearly fits with light-flows in both directions along each of the  $x$  and  $y$  axes. The projection of a light-flow  $c$  onto its own line of action ( $l_1$  &  $l_3$ ), in either direction, is clearly that same flow,  $c$ . Likewise  $l_5$  is clearly  $\sqrt{c^2 - v^2}$ , directly from Pythagoras' Theorem. The projected flow in the opposite direction would be a mirror image of this.

The complete ellipse is shown in Figure B1. We'll consider the implications of this ellipse over the next couple of pages.

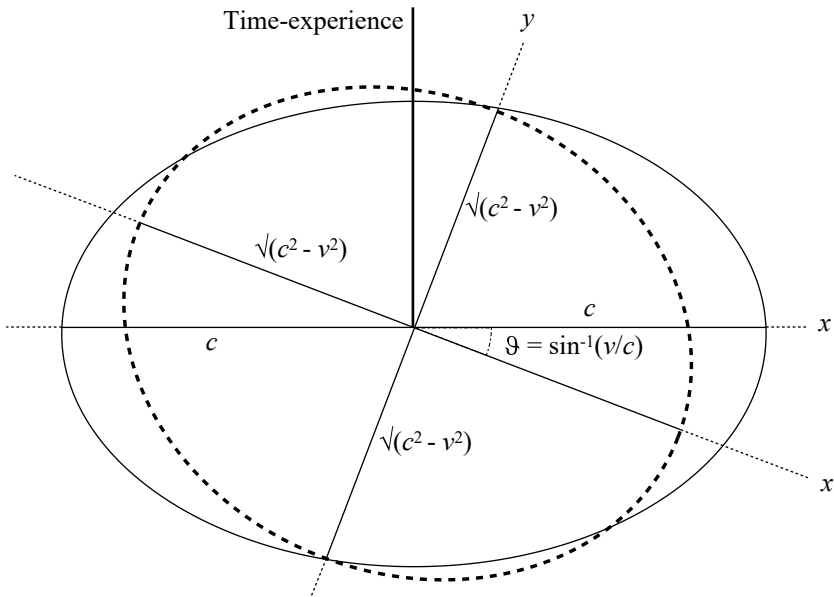
(B) Detailed spun-light mathematics of: Perceived Invariance of Light Speed.

Ok, so every light-flow interacting with an object moving at speed  $v$  does so as a projected flow whose magnitude is the length of the 'radial line', at that angle, of an ellipse with semi-major and semi-minor axes  $c$  and  $\sqrt{(c^2 - v^2)}$  respectively.

So what?

So this. The effect of the cyclic energy flows in our moving object is to tilt the effective  $x$ -axis (*i.e.* the direction of motion) out of the true spatial plane. The effect of this shift of perception is to foreshorten speeds in that direction, as shown in Figure 2 of Chapter 5. Speeds, or components of speed, in the spatial direction perpendicular to the direction of motion (*i.e.* in the  $y$ -direction) are unaffected.

The overall effect on the ellipse of projected flows is to tilt that ellipse around the  $y$ -axis through angle  $\vartheta$ , where  $v = c \sin \vartheta$ , as shown below in Figure B2. This maps the ellipse onto a new ellipse with the width (minor axis) unchanged but shortened length (major axis) along the  $x'$  line, as shown dotted in Figure B2.



**Fig. B2**

Elliptical envelope of light-flows in  $x$ - $y$  plane as incident on object moving at velocity  $v$  along  $x$ -axis, mapped onto  $x'$ - $y$  plane by moving object's perception

We've already seen how linear speed is the 'horizontal' component (along the spatial axis  $x$ ) of an object's energy flow, which is moving at light speed,  $c$ . As the speed of the object increases then the energy-flow line moves closer to the horizontal  $x$ -axis.

(B) Detailed spun-light mathematics of: Perceived Invariance of Light Speed.

At the ultimate point where the speed is equal to  $c$ , the flow is completely linear with no ‘vertical’ component - so it lies directly along the  $x$ -axis. This is light as we know it, moving freely through space, not bound up in a particle of matter, travelling along any spatial axis at full speed.

So the perception of that speed in either direction along the  $x$ -axis by our moving object will simply be the projection of  $c$  through angle  $\vartheta$  from the  $x$ -axis to the  $x'$ -axis. That projection will of course have a magnitude of:  $c \cos \vartheta$ .

$$\text{Since } \sin \vartheta = v/c, \cos \vartheta = \sqrt{1 - v^2/c^2} \quad [\cos^2 \vartheta = 1 - \sin^2 \vartheta]$$

$$\text{This gives us: } c \cos \vartheta = c\sqrt{1 - v^2/c^2} = \sqrt{c^2 - v^2}$$

In other words the semi-major axis,  $c$ , of our ellipse in the  $x$ - $y$  plane maps onto length  $\sqrt{c^2 - v^2}$  as the semi-major axis of our ellipse in the  $x'$ - $y$  plane - the set of possible light-flows as perceived by the moving object. This is shown in Figure B2, opposite. Also shown are the semi-minor axis lengths, unchanged at  $\sqrt{c^2 - v^2}$  - the same length as our new semi-major axis.

So we have an ellipse with both axes equal – which is of course a circle. This is the *envelope* of all possible effective approach vectors in the  $x$ - $y$  plane of light interacting with an object moving at speed  $v$  in that plane, adjusted to take account of that object’s own motion. Rotating that plane about the object’s line of motion, *i.e.* the  $x$ -axis, extends this result to cover every possible direction of approach.

So it turns out that, whatever the angle of approach, an object moving at speed  $v$  will experience light as travelling at speed  $\sqrt{c^2 - v^2}$ ?

Not quite. We haven’t yet taken into account the reduced-time-perception factor for that object due to its own speed. In Appendix A we established that as:

$$\sqrt{1 - \left( \frac{\text{linear speed}}{\text{flow speed}} \right)^2}$$

This is of course  $\sqrt{1 - (v/c)^2}$ , or  $\sqrt{c^2 - v^2} / c$ . Since this is the rate at which the clock ticks slower for the moving object, it will see more happening each second at the inverse rate of that factor:  $c / \sqrt{c^2 - v^2}$ .

**So the speed of light, from whatever direction, will be experienced by an object moving at speed  $v$  as:  $\sqrt{c^2 - v^2} \times \frac{c}{\sqrt{c^2 - v^2}} = c$**

**The energy flow model of matter shows *why* the speed of light is experienced as invariant by any object or observer, whatever their own speed.**